Unleashing Potentials
Shaping the Future

Chapter 4: Hypothesis Testing

Unit 3 - Testing the Difference Between Two Means
Paired Samples Test

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Outline

1. Testing the Difference Between Two Means: Using the *z*-Test
2. Testing the Difference Between Two Means of Independent Samples: Using the *t* Test
3. Testing the Difference Between Two Means: Dependent Samples

Learning Objectives

At the end of the session, student should be able to perform the following manually or using SPSS,

1. Testing the Difference Between Two Means: Using the *z*-Test
2. Testing the Difference Between Two Means of Independent Samples: Using the *t* Test
3. Testing the Difference Between Two Means: Dependent Samples

When the values are dependent, do a t test on the differences. Denote the differences with the symbol $\bar{d}$, the mean of the population differences with μ_d, and the sample standard deviation of the differences with s_d.

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} \quad ; \quad df = n - 1, \text{ where n is no. of pairs}$$

Example: Bank Deposits

A sample of nine local banks shows their deposits (in billions of dollars) 3 years ago and their deposits (in billions of dollars) today.

At $\alpha = 0.05$, can it be concluded that the average in deposits for the banks is greater today than it was 3 years ago? Use $\alpha = 0.05$.

Bank	1	2	3	4	5	6	7	8	9
3 years ago	11.42	8.41	3.98	7.37	2.28	1.10	1.00	0.9	1.35
Today	16.69	9.44	6.53	5.58	2.92	1.88	1.78	1.5	1.22

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Using Traditional Method:

Let,

μ_1 = mean deposit three years ago

μ_2 = mean deposit today

Hypothesis development:

$H_0: \mu_1 = \mu_2$	$H_0: \mu_1 < \mu_2$
$H_0: \mu_1 - \mu_2 = 0$	$H_0: \mu_1 - \mu_2 < 0$
$H_0: \mu_d = 0$	$H_0: \mu_d < 0$

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_d = 0$ and H_1: $\mu_d < 0$ (claim)

Step 2: Find the critical value.

The degrees of freedom = $n - 1 = 9 - 1 = 8$. The critical value for a left-tailed test with α=0.05 and d.f.=8 is equal to -1.860

TABLE F The *t* Distribution

	Confidence Intervals	80%	90%
	One tail, α	0.10	0.05
d.f.	Two tails, α	0.20	0.10
1		3.078	6.314
2		1.886	2.920
3		1.638	2.353
4		1.533	2.132
5		1.476	2.015
6		1.440	1.943
7		1.415	[illegible]
8		1.397	1.860
9		1.383	[illegible]
10		1.372	1.812

Step 4: Calculate the test statistic.

Use the following formula to determine the value of test value,

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$$; df = n – 1, where n is no. of pairs

$\mu_d = 0, \bar{d} = -1.0081, s_d = 1.937, n = 9$

$t = -1.674$

Step 4: Make the decision.

Do not reject the null hypothesis since the test value, –1.674, is greater than the critical value, –1.860.

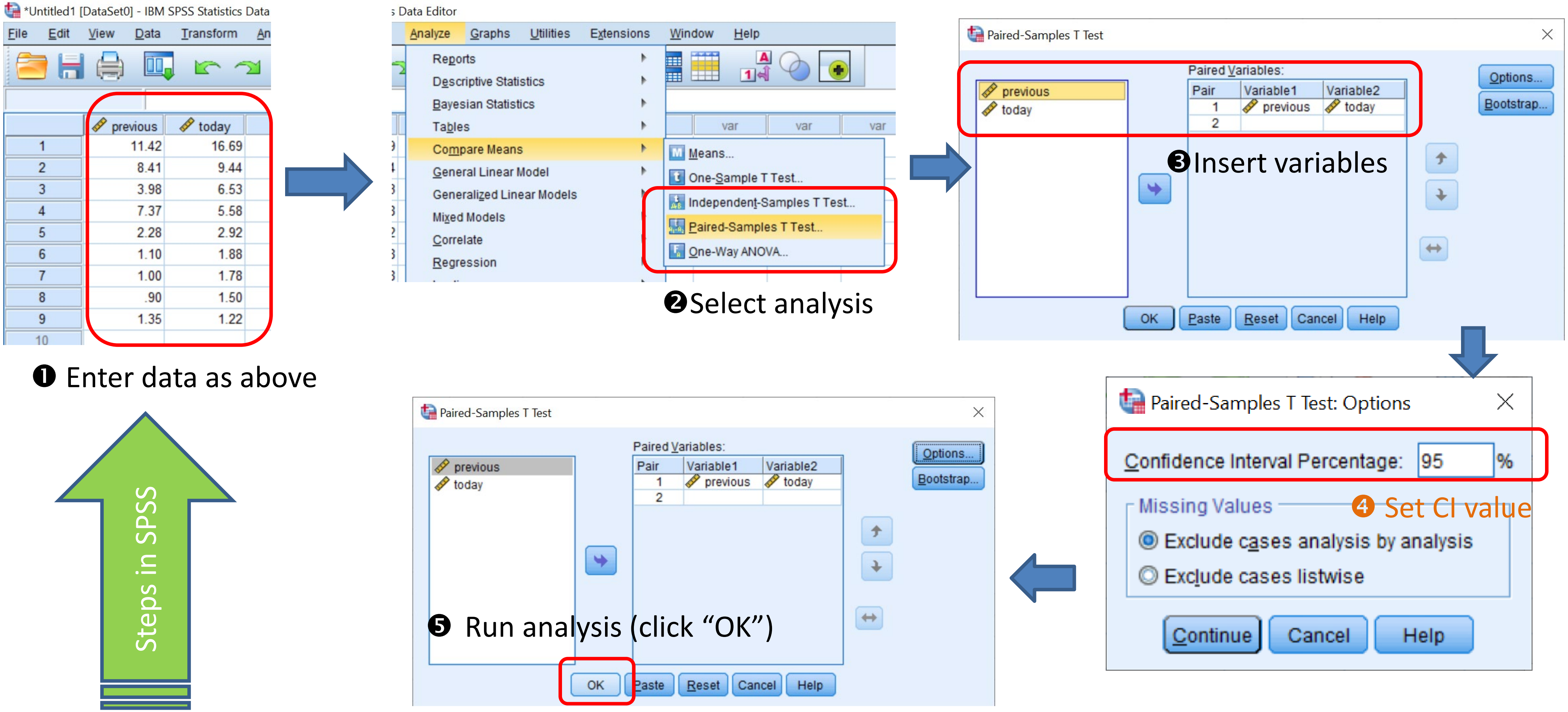
Since test value =-1.646 > critical value= -1.860, Fail to reject the null hypothesis.

Step 5: Summarize the results.

There is not enough evidence to show that the deposits have increased over the last 3 years.

Chapter 4 – Difference Between Two Means (Paired Samples)

SPSS Output



SPSS Output

3 years before	Today	D	D²
11.42	16.69	-5.27	27.77
8.41	9.44	-1.03	1.06
3.98	6.53	-2.55	6.50
7.37	5.58	1.79	3.20
2.28	2.92	-0.64	0.41
1.1	1.88	-0.78	0.61
1	1.78	-0.78	0.61
0.9	1.5	-0.6	0.36
1.35	1.22	0.13	0.02
		-9.73	40.54

$$\mu_D = \frac{-9.73}{9} = -1.08111$$

$$s_D = \sqrt{\frac{1}{9-1}\left(40.54 - \frac{(-9.73)^2}{9}\right)} = 1.9372$$

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	3 years ago	4.2011	9	3.91310	1.30437
	Today	5.2822	9	5.11490	1.70497

Paired Samples Test

$$\frac{Mean}{Std.\ Error\ Mean}$$

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	Upper			
Pair 1	3 years ago - Today	-1.08111	1.93728	.64576	-2.57024	.40802	-1.674	8	.133

$$\frac{1.93728}{\sqrt{9}}$$

From the **table Pair Samples Test**, there is a value of t which is -1.674 (Test value). You can obtain *t* value by taking mean difference divide by standard error mean :

t_{stat} = **-1.08111 /0.64576 = -1.67417 ≈ -1.674**

Using p-value method:

H_0: $\mu_d = 0$ and H_1: $\mu_d < 0$ (claim)

p-value =0.133. (p-value divide by 2 because this testing is one tailed test)

Since p-value =0.133/2 =0.0665 > $\alpha = 0.05$. Fail to reject H_0.

Conclusion: There is not enough evidence to show that the deposits have increased over the last 3 years.

Example: Cholesterol Levels

A dietitian wishes to see if a person's cholesterol level will change if the diet is supplemented by a certain mineral. Six subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. (Cholesterol level is measured in milligrams per deciliter.) Can it be concluded that the cholesterol level has been changed at $\alpha = 0.10$? Assume the variable is approximately normally distributed.

Subject	1	2	3	4	5	6
Before (X_1)	210	235	208	190	172	244
After (X_2)	190	170	210	188	173	228

Example: Cholesterol Levels (Solution)

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_d = 0$ and H_1: $\mu_d \neq 0$ (claim)

Step 2: Find the critical value.

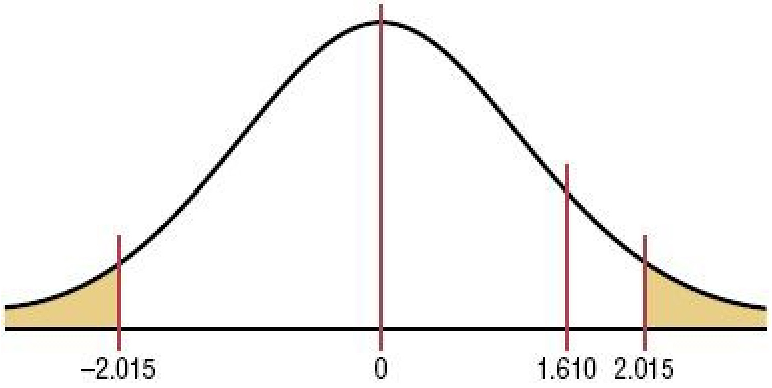
The degrees of freedom are 5. At $\alpha = 0.10$, the critical values are ±2.015.

Step 3: Compute the test value : $t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$; df = n – 1, where n is no. of pairs

$$t_{stat} = \frac{16.7 - 0}{25.4/\sqrt{6}} = 1.610$$

Step 4: Make the decision.

Do not reject the null hypothesis since $t_{stat} = 1.610 < t_{cv} = 2.015$



Step 5: Summarize the results.

There is not enough evidence to support the claim that the mineral changes a person's cholesterol level.

Example : Using Confidence Intervals method for hypothesis testing

Find the 90% confidence interval for the difference between the means for the data in previous example. The CI can be found using $\bar{d} \pm t_{\alpha/2} \frac{s_d}{\sqrt{n}}$

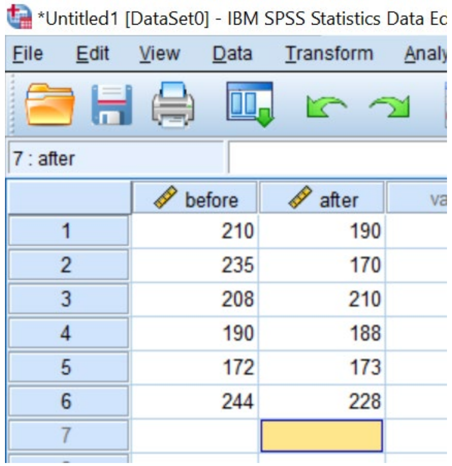
$$16.7 - 2.015 \cdot \frac{25.4}{\sqrt{6}} < \mu_D < 16.7 + 2.015 \cdot \frac{25.4}{\sqrt{6}}$$

$$16.7 - 20.89 < \mu_D < 16.7 + 20.89$$

$$-4.19 < \mu_D < 37.59$$

Since 0 is contained in the interval, the decision is **not to reject the null hypothesis** H_0: $\mu_d = 0$.

SPSSS Analysis Steps



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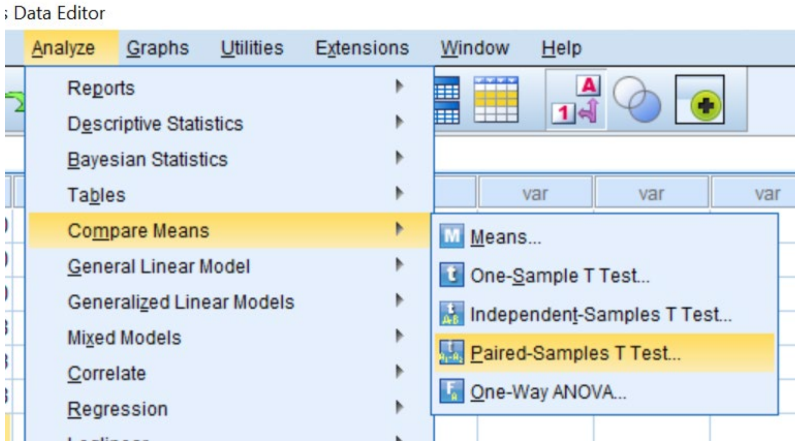

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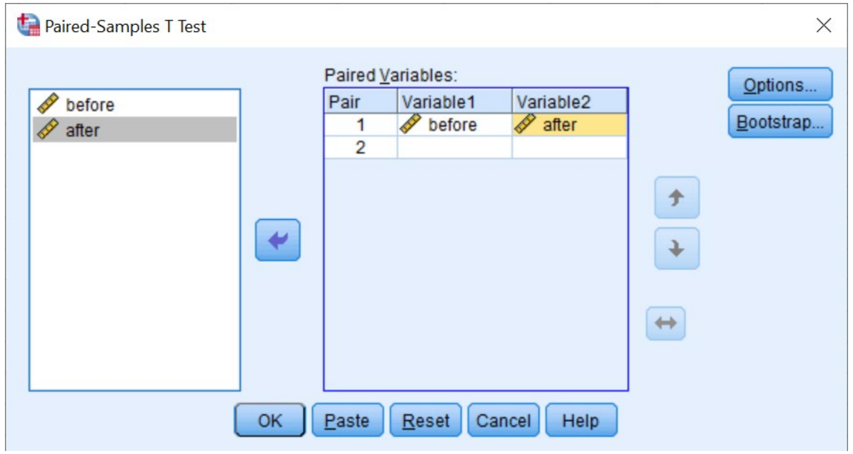

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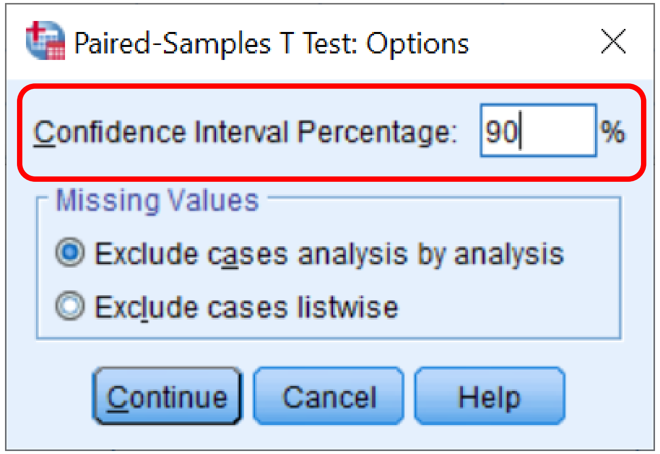

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Click 'OK' to run analysis

OUTPUT FROM SPSSS: (Using P value Method)

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Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before (X1)	209.83	6	26.940	10.998
	After (X2)	193.17	6	22.257	9.086

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
					90% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1	Before (X1) - After (X2)	16.667	25.390	10.366	-4.220	37.554	1.608	5	.169

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Using p-value method:

$H_0: \mu_d = 0$ and $H_1: \mu_d \neq 0$

Since p-value = 0.169 > α = 0.10, failed to reject H_0.

Conclusion: There is not enough evidence to support the claim that the mineral changes a person's cholesterol level.

For this example, we are using 3 methods for hypothesis testing:

1. Traditional method (compare test value and critical value)
2. Confident Interval method
3. *p*-value method

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END OF SLIDES

PRESENTATIONS

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